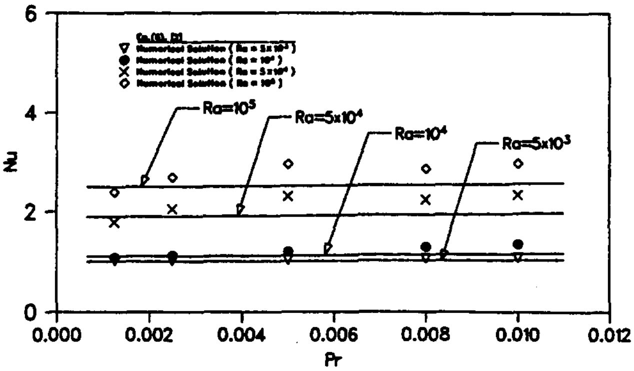

FIG. 5. Comparison of equation (6) and the present numerical solutions.

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Mixed convection experiments about a horizontal isothermal surface embedded in a water-saturated packed bed of spheres

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INTRODUCTION

NUMEROUS research articles have appeared in the literature in recent years in the area of convective heat transfer in fluid-saturated porous materials. The great majority of these articles deal with problems in natural convection or forced convection. The area of mixed convection, which constitutes the interface between natural and forced convection in porous media, has been, by comparison, largely overlooked.

One of the early investigations of combined free and forced

NOMENCLATURE

g	gravitational acceleration	U_∞	free-stream velocity
h	convective heat transfer coefficient	u	x-component velocity
K	permeability	x	horizontal Cartesian coordinate
k_e	effective thermal conductivity of porous medium	y	vertical Cartesian coordinate.
Nu_x	local Nusselt number	Greek symbols	
Pe_x	local Peclet number, $U_\infty x/\alpha_e$	α_e	effective thermal diffusivity of porous medium
Ra_x	local Rayleigh number, $g\beta\Delta TKx/\alpha_e\nu$	β	thermal expansion coefficient
T	temperature	δ_T	thermal boundary-layer thickness
T_w	wall temperature	ν	fluid kinematic viscosity
T_∞	free-stream temperature	ϕ	porosity.

convection is due to Combarnous and Bia [1] who studied, with the use of experiments and numerical computations, the effect of mean flow on the onset of convection in a porous medium bounded by isothermal planes. Cheng [2] analyzed the problem of mixed convection about inclined surfaces in a saturated porous medium. Solutions were obtained for the case where the free-stream velocity, and wall temperature distribution of the inclined surface, varied according to the same power function of distance. Later, Cheng [3] performed a boundary-layer analysis to obtain similarity solutions for mixed convection about a horizontal flat plate embedded in a saturated porous medium with aiding external flow and constant heat flux. Cheng [4] also applied integral methods to obtain local Nusselt numbers for mixed convection about a horizontal surface with constant heat flux and compared these values to the similarity solutions. Minkowycz *et al.* [5] extended Cheng's work by employing local non-similarity methods to obtain approximate solutions for the problem of mixed convection about a horizontal heated surface in a fluid-saturated porous medium with wall temperature being a power function of distance. Recently, Lai and Kulacki [6] used a non-Darcy flow model to present similarity solutions of mixed convection for the case of constant surface heat flux from horizontal impermeable surfaces in saturated porous media. With reference to experiments, very few studies have appeared in the literature [1, 7, 8].

In this paper, we present experimental results of mixed convection about a horizontal isothermal surface embedded in a water-saturated packed bed of spheres. The experimental set-up measures the approximate growth of the developing thermal boundary-layer thickness and the local heat flux at the surface. A scale analysis of the mixed convection problem is formulated and reported. The governing parameter for mixed convection about a horizontal isothermal surface in a porous medium is found to be $Ra_x^{1/3}/Pe_x^{1/2}$.

EXPERIMENTAL APPARATUS AND PROCEDURES

The experimental apparatus and procedures employed in this investigation are almost identical to ref. [9]. The only change is the substitution of a heated bottom plate test section. No further details are given here for brevity. The system provided a means of accurately measuring the fluid flow and heat flux for an experiment whose main purpose was to obtain mixed convection results for a horizontal isothermal flat plate inside a water-saturated porous medium.

The fluid properties (e.g. density, specific heat, viscosity and thermal conductivity) were based on the inlet temperature and were calculated using regression curve fits. The standard error of the fluid property estimates was negligible. The experimental effective thermal conductivity, k_e, was computed from empirical formulas found in the literature [10]. The experimental thermal boundary-layer thickness, δ_T was approximated by comparing the free-stream (inlet) temperature and the vertical thermocouple readings. The experimental local Nusselt number was determined as follows:

$$Nu_x = \left(\frac{dT}{dy}\right)_{y=0} \frac{x}{T_w - T_\infty} \tag{1}$$

where x is the horizontal spatial coordinate from the heated test section entrance. The local wall temperature gradient, $(dT/dy)_{y=0}$ was evaluated by using the plate temperature and the reading of the thermocouple in the porous medium nearest to the wall (1.5 mm from the surface) at each station. The local Nusselt number was also evaluated by calculating and non-dimensionalizing the heat flux (power) produced by the strip heaters at the wall. The product of the measured current and voltage determined the heat flow across the transfer surface. Good agreement (within 5%) between the two methods was found.

SCALING ANALYSIS

Figure 1 shows a schematic of the problem of interest, namely mixed convection in a water-saturated bed of glass spheres adjacent to a horizontal isothermal surface. The free-stream fluid temperature at the heated section inlet is isothermal and designated as $T_\infty (T_w > T_\infty)$. The coordinate system is also defined in Fig. 1, where x and y are Cartesian coordinates in the horizontal and vertical directions with the positive y axis pointing vertically toward the porous medium. The origin of the coordinate system is at the start of the heated wall section. We assume that: (i) the fluid and the porous medium are in local thermal equilibrium; (ii) the properties of the fluid and the porous medium are constant; and (iii) the Boussinesq approximation can be employed.

Relative to the above conditions, geometry and two-dimensional coordinate system, the steady state boundary-

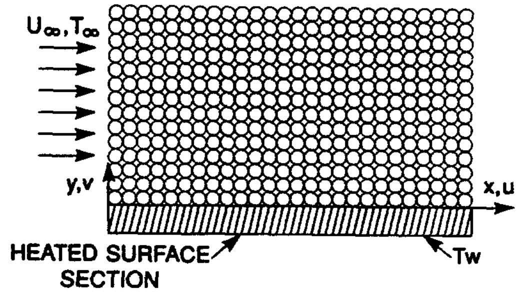

FIG. 1. Schematic of problem: mixed convection in a water-saturated packed bed of spheres adjacent to a horizontal isothermal surface.

layer governing equations for the problem of pure free convection in a saturated porous medium were used [11]. Defining $\Delta T = T_w - T_\infty$ as the scale for the temperature differences encountered in the boundary-layer region and making use of the slenderness of the thermal boundary layer ($\delta_T \ll x$), we can show that the scale for the horizontal velocity resulting from the momentum equation is

$$u \approx O\left(\frac{K}{\nu} g\beta\Delta T \frac{\delta_T}{x}\right). \tag{2}$$

Similarly, the energy equation yields the following scale for the horizontal velocity:

$$u \approx O\left(\alpha_e \frac{x}{\delta_T^2}\right). \tag{3}$$

Equations (2) and (3) combined yield the scale for the thickness of the thermal boundary layer

$$\frac{\delta_T}{x} \approx O(Ra_x^{-1/3}). \tag{4}$$

The scale for the local Nusselt number is then

$$Nu_x = \frac{hx}{k_e} \approx O(Ra_x^{1/3}). \tag{5}$$

The corresponding scale for Nu_x and δ_T/x for the problem of purely forced convection have been derived in Bejan [11]

$$\frac{\delta_T}{x} \approx O(Pe_x^{-1/2}) \tag{6}$$

$$Nu_x \approx Pe_x^{1/2}. \tag{7}$$

Next, in order to decide which mechanism dominates over the other (natural convection over forced convection or vice vera), it is enough to compare the respective scales of the thermal boundary-layer thickness. For example, forced convection is dominant if $\delta_T|_{\text{forced convection}} \ll \delta_T|_{\text{free convection}}$ or

$$\frac{Ra_x^{1/3}}{Pe_x^{1/2}} \ll O(1). \tag{8}$$

Similarly, natural convection dominates if

$$\frac{Ra_x^{1/3}}{Pe_x^{1/2}} \gg O(1). \tag{9}$$

Finally, as the group $Ra_x^{1/3}/Pe_x^{1/2}$ approaches unity either from above or from below, the mechanisms of natural and forced convection coexist and mixed convection takes place. The governing parameter, $Ra_x^{1/3}/Pe_x^{1/3}$ will be used later in the study to correlate the experimental data.

DISCUSSION OF RESULTS

In this section, the results of our mixed convection experiment are reported and compared to the theoretical solutions existing in the literature. To make possible such comparisons key theoretical results for forced and free convection will be transferred here. All of the details of the analysis can be found in the literature [2, 5, 11–14].

For the pure forced convection case of the two-dimensional system of Fig. 1, the Darcy model solution yields [2, 5, 11]

$$Nu_x = 0.564 Pe_x^{1/2}. \tag{10}$$

Vafai and Tien [12] report the following approximation for the thermal boundary-layer thickness δ_T for the case of uniform parallel flow through a saturated porous medium adjacent to a horizontal isothermal impermeable boundary

$$\delta_T = \frac{7.0x}{(\phi\, Pe_x)^{1/2}}. \tag{11}$$

The analogous expressions for δ_T and Nu_x for purely natural convection from a horizontal plate are [13, 14]

$$\delta_T = \frac{5.5x}{Ra_x^{1/3}} \tag{12}$$

$$Nu_x = 0.430 Ra_x^{1/3}. \tag{13}$$

Figure 2 presents the dependence of the dimensionless thermal boundary-layer thickness $\delta_T Pe_x^{1/2}/x$ on the governing parameter $Ra_x^{1/3}/Pe_x^{1/2}$. Since the temperature measurements were taken at selected horizontal and vertical locations, it was estimated that the thermal boundary-layer thickness values were accurate to within 10%. The theoretical model for the pure forced convection asymptote is reported by Vafai and Tien [12], equation (11), while the pure free convection asymptote predicted by Cheng and Chang [13] is equation (12). The experimental data clearly show a decrease in the dimensionless thermal boundary-layer thickness as the governing parameter $Ra_x^{1/3}/Pe_x^{1/2}$ increases, or alternatively, the thermal boundary-layer thickness is increasing as the Peclet number decreases. Comparing our experimental results with the convection asymptotes, we find that the majority of our data fall within the regime between pure forced and mixed convection. The theoretical model underpredicts the dimensionless thermal boundary-layer thickness at small values of $Re_x^{1/3}/Pe_x^{1/2}$. The theoretical model of pure convection is untested, but the experimental data do show a trend of decrease in $\delta_T Pe_x^{1/2}/x$ similar to the pure free convection asymptote. The set of data in Fig. 2 is correlated by

$$\frac{\delta_T}{x} Pe_x^{1/2} = 5.43\left(\frac{Ra_x^{1/3}}{Pe_x^{1/2}}\right)^{-0.34}. \tag{14}$$

Figure 3 reports our findings for the local heat transfer coefficient along the horizontal isothermal surface. The dependence of $Nu_x/Pe_x^{1/2}$ on the single parameter $Ra_x^{1/3}/Pe_x^{1/2}$ is stressed. The estimated overall error for the experimental values of the Nusselt number was 5.5%. The theoretical predictions of Darcy flow forced convection solution, equation (10), reported by refs. [2, 5, 11] and the pure free convection, equation (13), by refs. [13, 14] are presented as asymptotes for comparison. The experimental results show that as the value of $Ra_x^{1/3}/Pe_x^{1/2}$ decreases the value of $Nu_x/Pe_x^{1/2}$ decreases or the Nu_x value increases. Again, we do not find good agreement between the experimental data and the forced convection model at small values of $Ra_x^{1/3}/Pe_x^{1/2}$. The Darcy model overpredicts the heat transfer in the forced convection limit. The set of data for the mixed convection region is correlated by

$$\frac{Nu_x}{Pe_x^{1/2}} = 1.15\left(\frac{Ra_x^{1/3}}{Pe_x^{1/2}}\right)^{0.29} \tag{15}$$

Even though we have no data for high values of

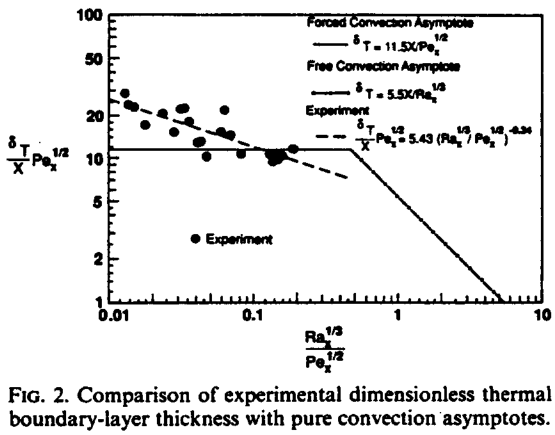

FIG. 2. Comparison of experimental dimensionless thermal boundary-layer thickness with pure convection asymptotes.

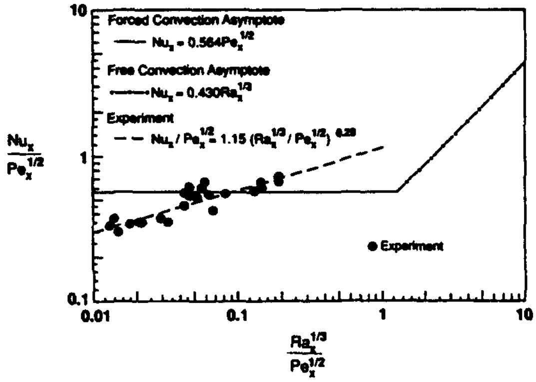

FIG. 3. Comparison of experimental Nusselt number with pure convection asymptotes.

$Ra_x^{1/3}/Pe_x^{1/2}$ because of experimental set-up limitations, the trend shown in Fig. 3 appears promising, i.e. the Darcy model may predict the value of Nu_x in the natural convection limit better than it does in the forced convection limit.

CONCLUDING REMARKS

An experimental investigation of the problem of mixed convection in a packed bed of glass spheres about a horizontal isothermal surface was presented. The experimental results document the dependence of the growth of the thermal boundary-layer and the variation of the local heat flux (reported by the local Nusselt number) on the parameter $Ra_x^{1/3}/Pe_x^{1/2}$. This parameter was obtained from scaling analysis of the Darcy model and it describes the transition between forced and natural convection. Several comparisons were made between the experimental findings and existing theoretical results for pure forced and pure free convection. It was found that the experimental results lie in the forced convection end of the mixed convection range. The Darcy model appears to overpredict heat transfer in the forced convection limit. Two correlations were obtained for the thermal boundary-layer and the local Nusselt number in the mixed convection regime.

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